THE RIVER CAM BRIDGE

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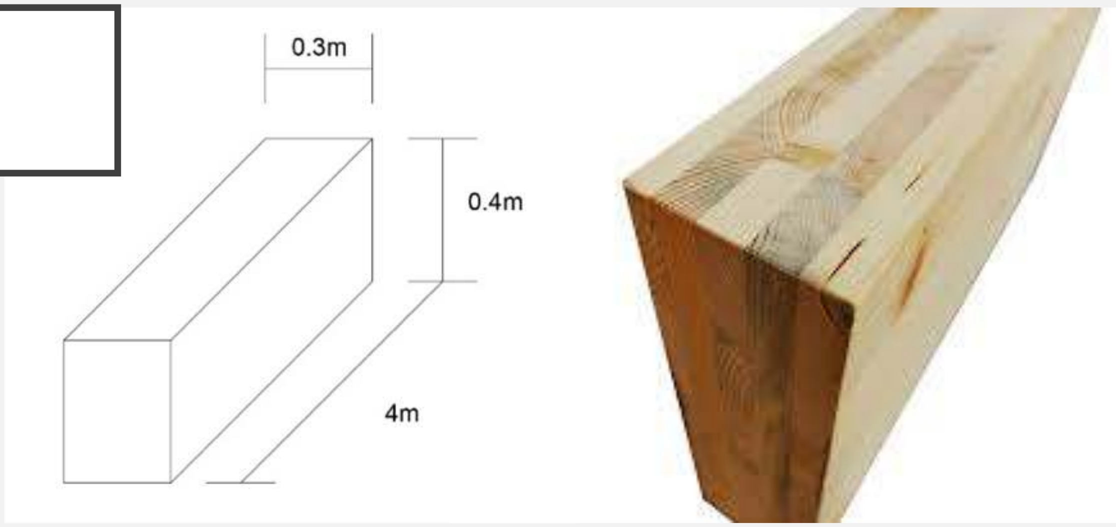
LOCATION & MATERIALITY

- Located in a Northern European setting, specifically South-East England. The model is placed in a park near Queen's College, Cambridge and bridges the River Cam.
- Pratt truss system to avoid the use of additional external supports.
- Designed with wood so that it could be constructed sustainably



- Material = Cross-Laminated Timber (Dolomiti Productions)

Elastic Modulus (E) = 11500 MPa
Shear Modulus (G) = 650 MPa
Design strength in Tension *(Sten)* = 19.2 MPa
Design strength in Compression (*Scomp*) = 24 MPa
Density = 385 *kgm3*

Beam Dimensions:
Main beams (x2) = = 0.4m x 0.3m x 16m = 3.84 m^3
Secondary beams (x29) = 0.4m x 0.3m x 4m = 13.92 m^3
Floor panels (64m^2) = 0.05m x 1m x 4m = 12.8 m^3
Major braces (x 10) = 0.4m x 0.3m x 5.66m = 6.79 m^3
Minor bracing (x6) = 0.17m x 0.17m x 2.34m = 0.41 m^3

Total Bridge Weight:
Total mass = 37.76m^3 of cross-laminated timber
37.76 x 385 ≅ $14{,}500\ kg$ total bridge weight
Self-weight load = ${}^{14{,}500kg}/_{64m^2}\ (bridge\ surface\ area)$ =
226,6 ${}^{kg}/_{m^2}$ ≅ $2{,}266\ {}^{KN}/_{m^2}$
Self-weight (Eurocodes) ≅ 2.22 ${}^{KN}/_{m^2}$

Total Bridge Loads: 6.82 ${}^{KN}/_{m^2}$

STATIC SCHEME

Secondary Beams:

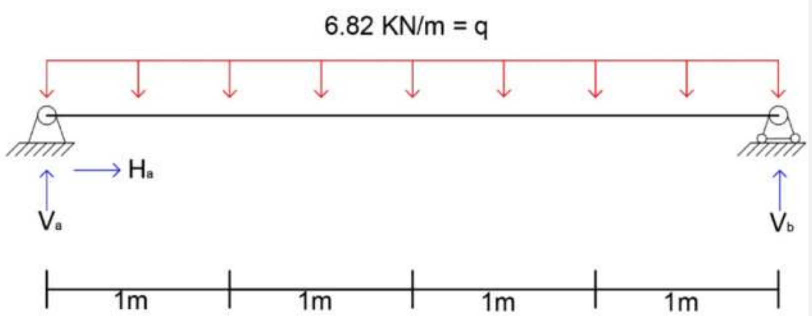
- The secondary beams are those that directly support the floorboards of the bridge. They are 4 metres long and spaced a metre apart from one another for the bridges entire span. This means that each secondary beam ends up supporting $4m^2$ of distributed load, with a support at each extremity.

External Reactions:

$$\sum F_x = 0 = H_a$$

$$\sum F_y = 0 = V_a + V_b - 6.82 \ x \ 4$$

$$V_a = V_b = 6.82 \ x \ 2 = 13.64 \ KN$$

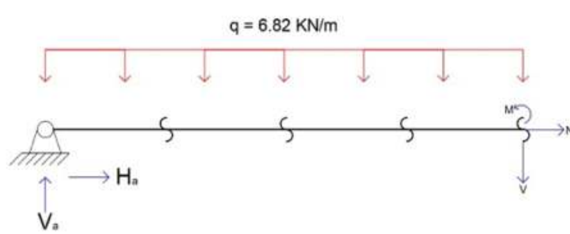


Internal Reactions:

N = 0

$$\sum F_y = 0 = V_a - V_{\square} - qL$$
$$= 13.64 - V - 6.82 \ x \ L$$

$$\sum M_a = 0 = M - V_{\square} - VL - 6.82 \ x \ \frac{L^2}{2}$$



@L = 0m:

- V = 13.64 - 6.82 x 0 = 13.64KN
- M = 0

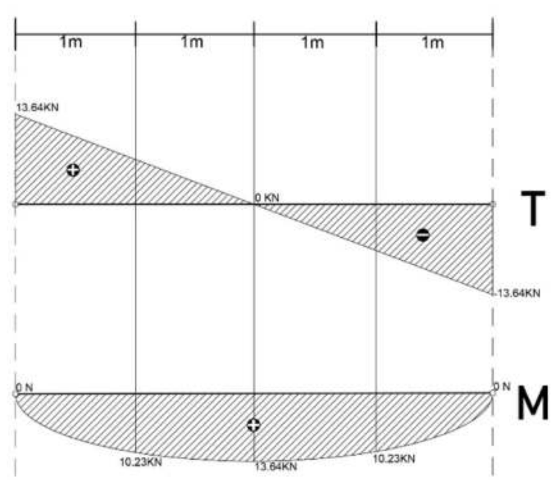
@L = 1m:

- M = 6.82 + $\frac{6.82}{2}$ = 10.23 KN

@L = 2m:

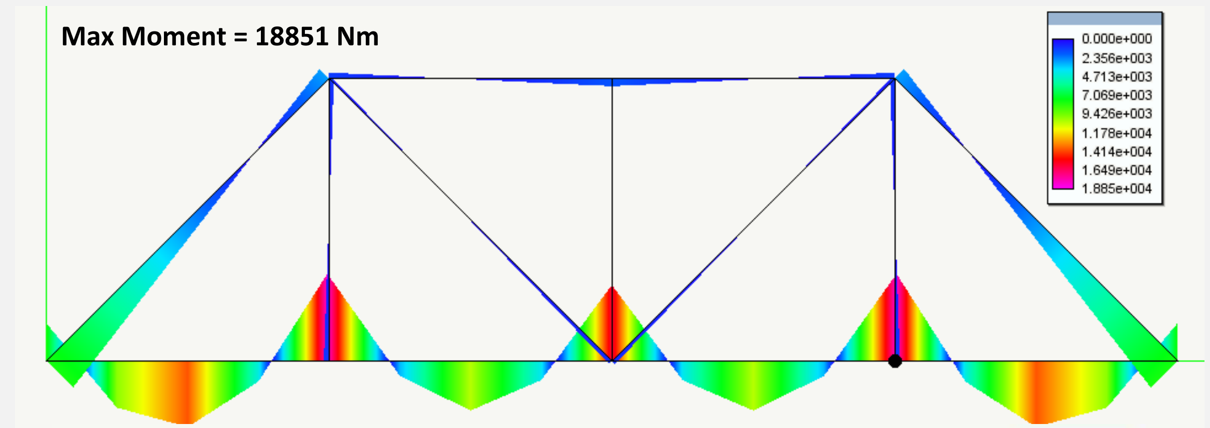
- V = 13.64 - 6.82 x 2 = 0 KN
- M = 13.64 x 2 - $\frac{6.82 \ x \ 2^2}{2}$ = 13.64 KN

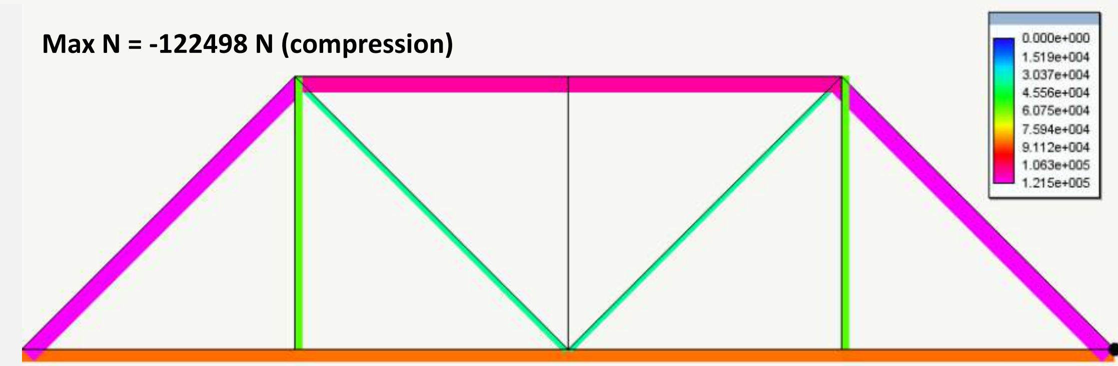
SYMMETRICAL SYSTEM

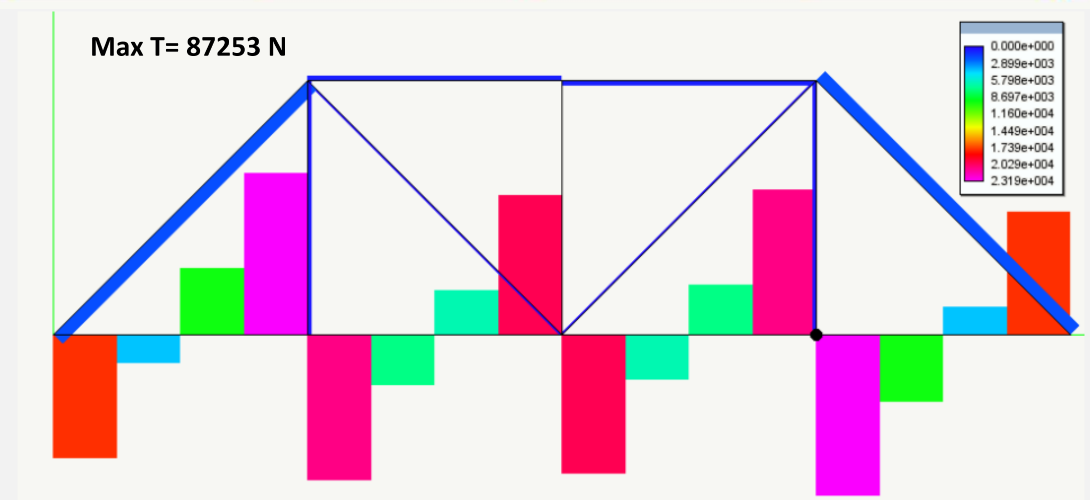


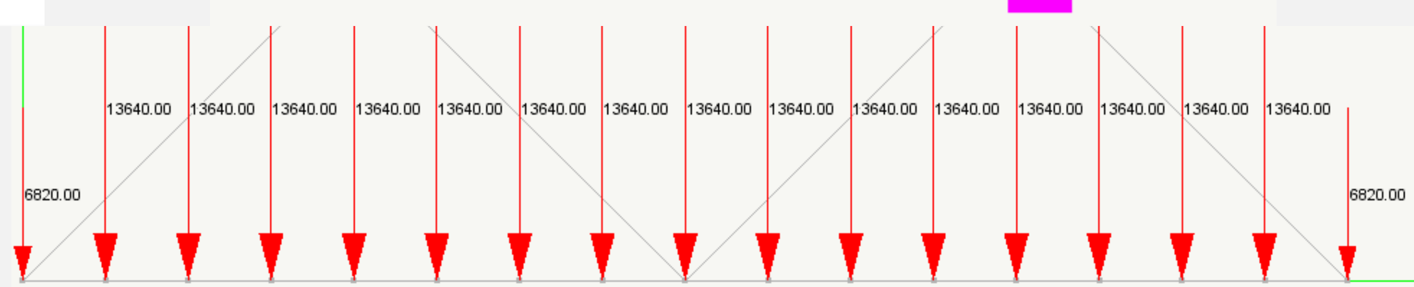
Primary Beams (Nolian Truss system):

- The two primary beams run either side of the bridge and take the loads from the secondary beams. Because of this, they have a point-load at every one metre interval equal to half the load that the secondary beam supports. The two beams at either end only support half as much load, as they only support half as much floor area.



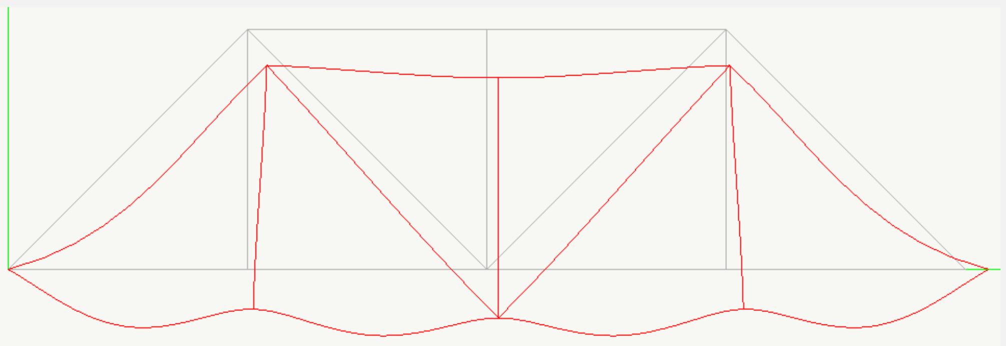






DEFORMATION

It is clear from this diagram, how necessary the truss system is to support the loads on the primary beams, as even with the additional supports, this is the area of maximum deflection. Fortunately, our scheme assures that this deformation remains within acceptable limits.



Critical Deflection =16m/400 = 0.04m
Maximum Deflection = 0.003 m
Max deflection < Critical Deflection:
deformation occurs within acceptable parameters

SLENDERNESS

The slenderness ratio is a measure of how likely a member is to buckle due to compression by comparing the thinnest dimension of the beam's cross section to the overall length of the beam. Eurocode guidelines suggest that timber beams that are used for load-bearing purposes should have a slenderness ratio below 50, regardless of the size of the loads present.

Radius of Gyration $(R_g) = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{0{,}009m^4}{0{,}06m^2}} = 0{,}1225m$

Slenderness $= \frac{L_{beam}}{R_g} = \frac{5{,}66m}{0{,}1225m} = 46.2$

46.2 < 50: suitable for sustaining live loads

CRITICAL LOAD

It was when assessing the critical load for our original oak structure that we discovered that it would potentially fail.

The issue was that with a smaller cross section our moments of inertia were not satisfactory to withstand the compressive forces in our vertical braces. This had the effect of making our maximum normal force higher, while also making our critical loads lower.

N_{max} = 122.5 KN

Maximum Stress $(\sigma_z^{max}) = \frac{\pi^2 EI}{L^2}$

E = Elastic Modulus = 11.5GPa = 1150000 KN/m²

I = Moment of Inertia: $I_x = \frac{bh^3}{12} = \frac{0{,}3 \ x \ 0{,}4^3}{12} = 0{,}0016m^4$

$I_y = \frac{b^3h}{12} = \frac{0{,}3^3 x \ 0{,}4}{12} = 0{,}0009m^4$

$N_{x,critical} = \frac{\pi^2 \ x \ 1150000 \ x \ 0{,}0016}{5{,}66^2}$ = 5669 KN

$N_{y,critical} = \frac{\pi^2 \ x \ 1150000 \ x \ 0{,}0009}{5{,}66^2}$ = 3189 KN

$N_{max} < N_{y,critical} < N_{x,critical}$: Critical loads not reached

STRENGTH DESIGN

While cross-laminated timber has good design strength in tension and compression, Eurocode guidelines also ask that wooden load-bearing structures apply two multipliers to the design strengths. The design multiplier (f) is to offset the risk of a lower actual material strength due to things like defects. A k-multiplier is also present, to offset the risk of the wood splitting overtime.

$$\sigma_z^{max} = \frac{N_z}{A} + \frac{M_x}{I_x} Y_{max} = \frac{122{,}5KN}{0{,}12m^2} + \frac{18{,}851KNm}{0{,}0016m^4} \cdot 0{,}2m = 3.38 \ MPa$$

Design strength in Tension (S_{ten}) = 19.2 MPa

Design strength in Compression (S_{comp}) = 24 MPa

Design multiplier (f) = 0.8 (as specified by Eurocodes)

Risk of splitting over lifespan (k) = 0.6 (for long-term structures)

$\sigma_{lim} = S \cdot f \cdot k$

$\sigma_{lim,tension} = S \cdot f \cdot k = 19.2 \cdot 0.8 \cdot 0.6 = 9.22 \ MPa$

$\sigma_{lim,compression} = S \cdot f \cdot k = 24 \cdot 0.8 \cdot 0.6 = 11.52 \ MPa$

3.38 MPa < 9.22 MPa < 11.52 MPa: Material strong enough in tension and compression

HORIZONTAL FORCES

Our bridge is braced by much thinner beams to resist it against horizontal forces, so it is necessary to confirm that these minor braces are adequate to the task.

- The predicted horizontal force is 15% of the overall vertical load on the bridge, which equates to approximately $1\ {}^{KN}/_{m^2}$.

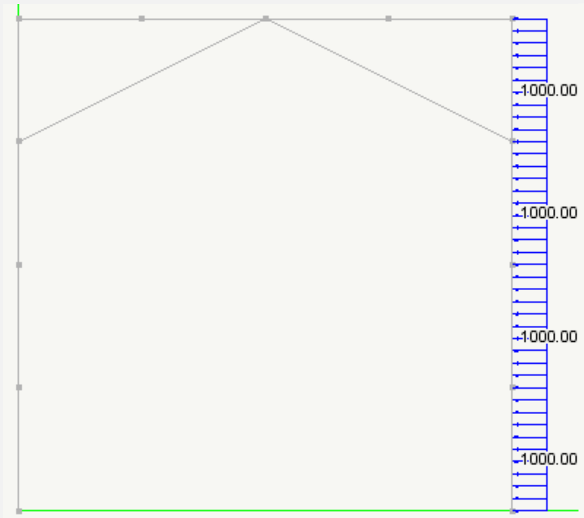
$6.82 \times 0.15\ \cong 1\ {}^{KN}/_{m^2}$

Max M = 3,783 KNm

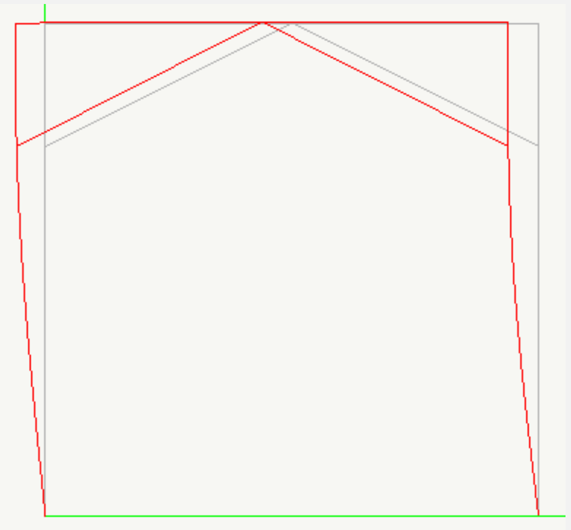
Max N = 5,695 KN

Max T = 4,338 KN

Max Displacement = 0.0019m < 4/400 = 0.01m



Static scheme



Displacement

Minor Bracing Critical Loads

N_{max} = 5.7 KN

Maximum Stress $(\sigma_z^{max}) = \frac{\pi^2 E I_x}{L^2}$

E = Elastic Modulus = 11.5GPa = 1150000 KN/m²

I = Moment of Inertia: $I_x = I_y = \frac{bh^3}{12} = \frac{17 \times 17^3}{12} = 0{,}0000669 m^4$

$N_{critical} = \frac{\pi^2 \times 115000 \times 0{,}0000669}{2{,}34^2}$ = 139 KN

$N_{max} < N_{critical}$: Critical loads not reached

Minor Bracing Slenderness

Radius of Gyration $(R_g) = \sqrt{\frac{I}{A}} = \sqrt{\frac{0{,}0000669 m^4}{0{,}0289 m^2}}$ = 0,0481m

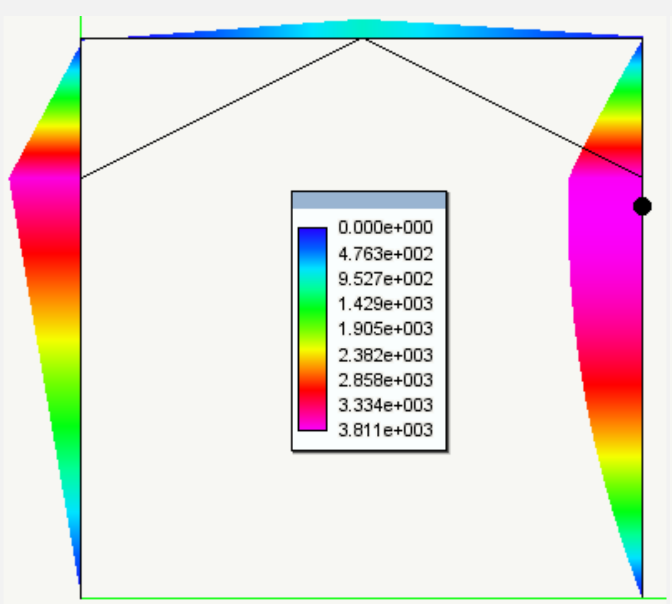
Slenderness (λ) = $\frac{L_{beam}}{R_g} = \frac{2{,}34m}{0{,}0481m}$ = 48.7

48.7 < 50: suitable for sustaining live loads

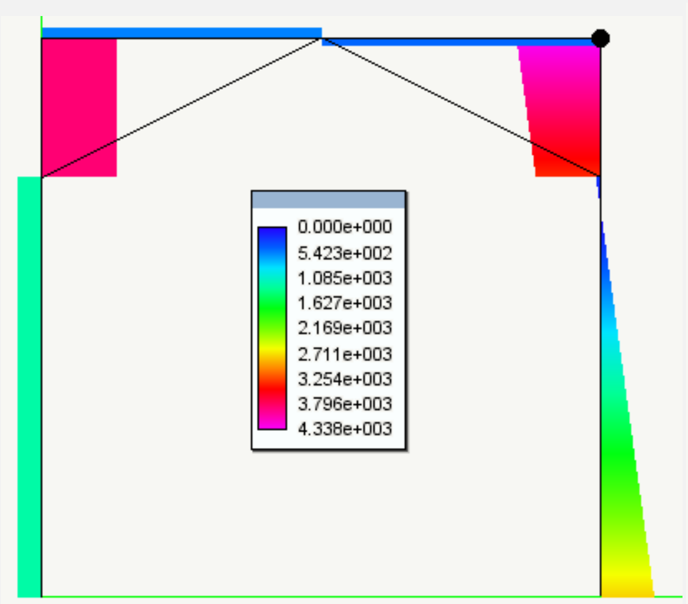
Minor Bracing Strength Design

$$\sigma_z^{max} = \frac{N_z}{A} + \frac{M_x}{I_x} Y_{max} = \frac{5.7KN}{0{,}0289m^2} + \frac{0{,}038KNm}{0{,}0000669m^4} \cdot 0{,}085m = 0.25\ MPa$$

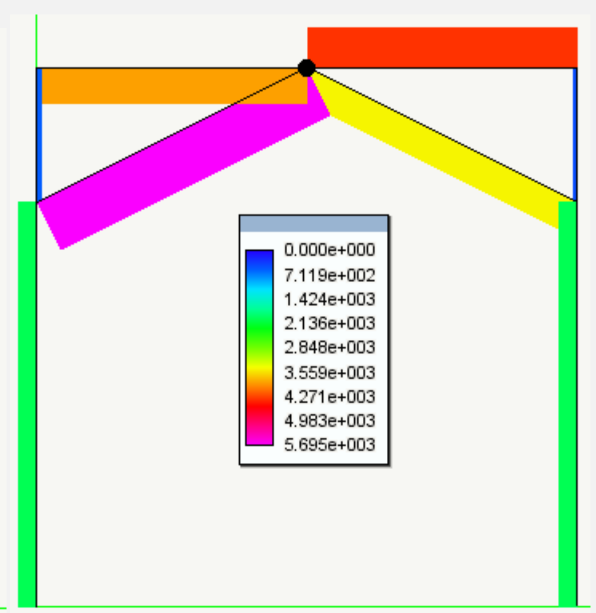
0.25 MPa < 9.22 MPa < 11.52 MPa: Material strong enough in tension and compression



Bending Moment (M)



Shear Forces (T)



Normal Forces (N)

THANK YOU